Math 10 Lecture Videos

Section 4.1: Solving Systems of Linear Equations by Graphing

PAUL ANDREW GORGONIO

OBJECTIVES:

1. Determine whether an ordered pair is a solution of a linear system.
2. Solve systems of linear equations by graphing.
3. Use graphing to identify systems with no solution or infinitely many solutions.

Objective 1: **Determine whether an ordered pair is a solution of a linear system.**

An equation of the form *Ax + By = C* is a line when graphed. Two such equations are called a **system of linear equations**.

A **solution** to a system of linear equations is an **ordered pair** that **satisfies both equations** in the system.

For example, the ordered pair (2,1) satisfies the system:

$\mathbf{3x + 2y = 8}$	$\mathbf{4x - 3y = 5}$
$3(2) + 2(1) = 8$	$4(2) - 3(1) = 5$
$6 + 2 = 8$	$8 - 3 = 5$
$\mathbf{8 = 8}$	$\mathbf{5 = 5}$

Objective 1: Determine whether an ordered pair is a solution of a linear system.

Systems of Linear Equations

Since two lines may intersect at **exactly one point**, may **not intersect at all**, or **may intersect at every point**, it follows that a system of linear equations will have ***exactly one solution***, will have ***no solution***, or will have ***infinitely many solutions***.

Objective 1: Determine whether an ordered pair is a solution of a linear system.

Example 1: Determine whether (3,2) is a solution of the system.

$$-3x + 6y = 10$$
$$5x - 8y = -2$$

Let's try using one of the equations to check:

$-3x + 6y = 10$
$-3(3) + 6(2) = 10$
$-9 + 12 = 10$
$3 \neq 10$

$5x - 8y = -2$
$5(3) - 8(2) = -2$
$15 - 16 = -2$
$-1 \neq -2$

Since the results are false, (3,2) is NOT a solution of the system.

Objective 1: Determine whether an ordered pair is a solution of a linear system.

Example 2: Determine whether (4,-7) is a solution of the system.

$$x + y = -3$$
$$2x + y = 1$$

Let's try using one of the equations to check:

$x + y = -3$
$4 + (-7) = -3$
$-3 = -3$

$2x + y = 1$
$2(4) + (-7) = 1$
$8 + (-7) = 1$
$1 = 1$

Since the results are true, (4,-7) is a solution of the system.

Objective 2: Solve systems of linear equations by graphing.

Solve Systems of Two Linear Equations in Two Variables, *x* and *y*, by Graphing
1. Graph the first equation.
2. Graph the second equation on the *same set of axes*.
3. If the lines representing the two graphs intersect at a point, determine the coordinates of this point of intersection. The ordered pair is the solution to the system.
4. Check the solution in *both* equations.

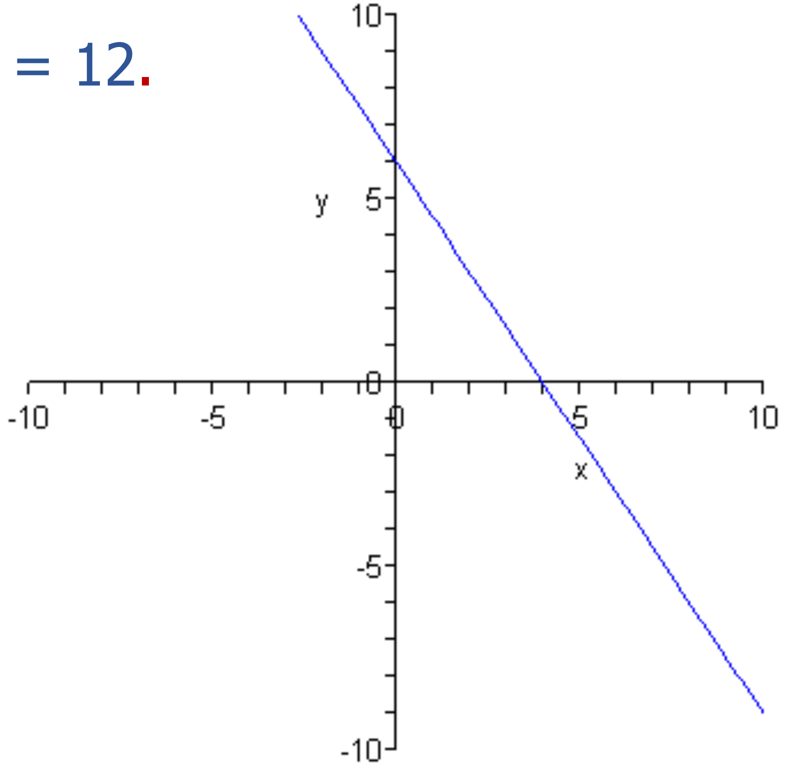
NOTE: In order for this method to be useful, you must graph the lines *very accurately*.

Objective 2: Solve systems of linear equations by graphing.

Example 1: Solve the system by graphing:

$$3x + 2y = 12$$
$$2x - y = 1$$

1. Graph the first line. Find x and y intercepts of $3x + 2y = 12$.



X – Intercept (let y = 0)
$3x + 2y = 12$
$3x + 2(0) = 12$
$3x = 12$
$x = 4$

Y – Intercept (let x = 0)
$3x + 2y = 12$
$3(0) + 2y = 12$
$2y = 12$
$y = 6$

The x-intercept is 4, while the y-intercept is 6.

Objective 2: Solve systems of linear equations by graphing.

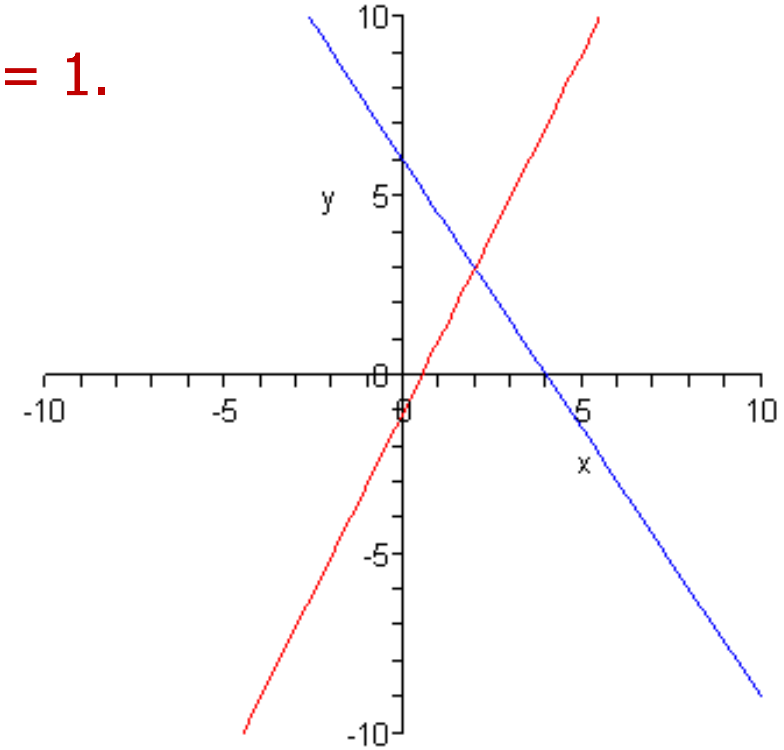
Example 1: Solve the system by graphing:

3x + 2y = 12

2x – y = 1

2. Graph the second line. Find x and y intercepts of 2x - y = 1.

X – Intercept (let y = 0)	**Y – Intercept (let x = 0)**
2x – y = 1	2x – y = 1
2x – 0 = 1	2(0) – y = 1
2x = 1	-y = 1
x = 0.5	y = -1



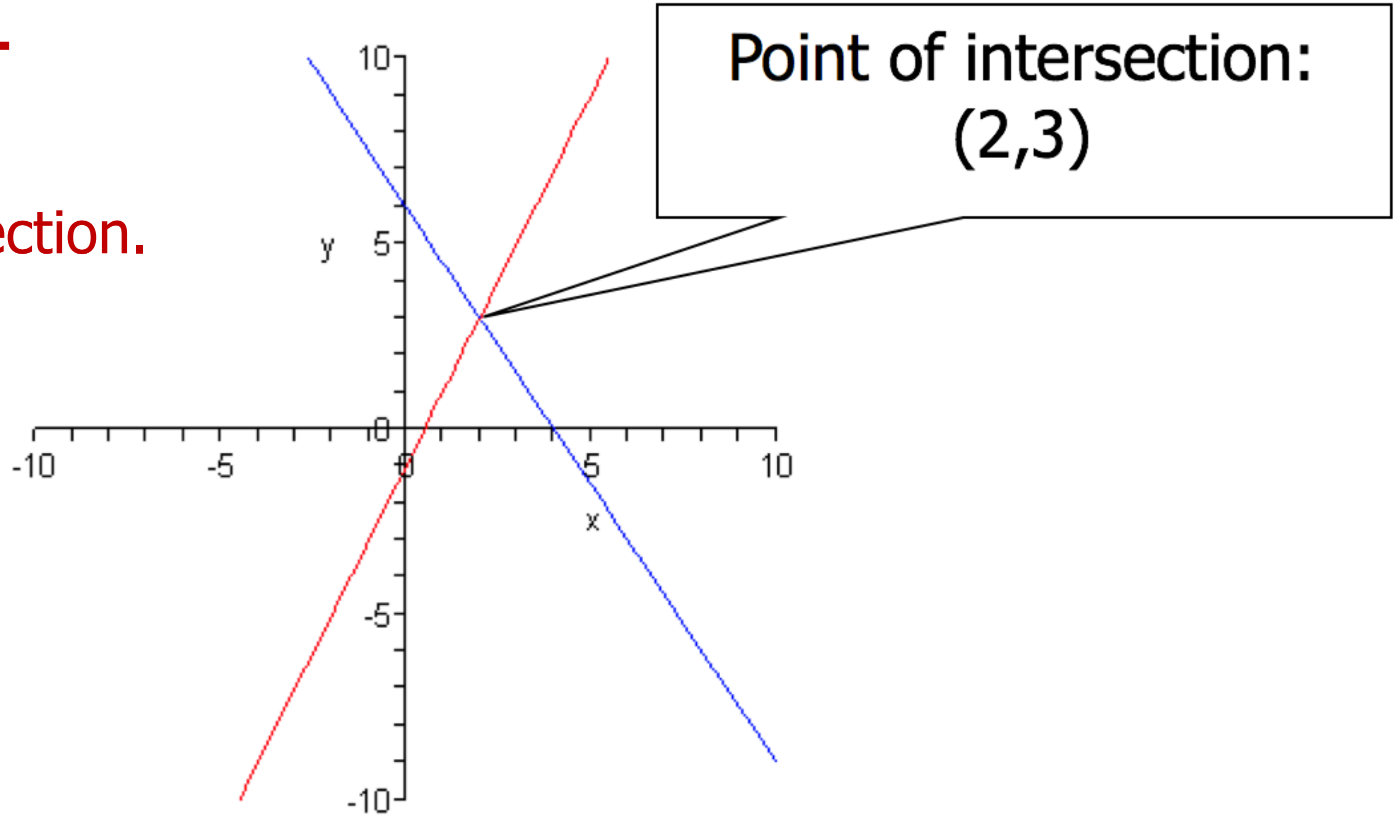
The x-intercept is 0.5, while the y-intercept is -1.

Objective 2: Solve systems of linear equations by graphing.

Example 1: Solve the system by graphing:

$$3x + 2y = 12$$
$$2x - y = 1$$

3. Find the coordinates of intersection.



Objective 2: Solve systems of linear equations by graphing.

Example 1: Solve the system by graphing:

$$3x + 2y = 12$$
$$2x - y = 1$$

4. Check the proposed solution in each equation

Equation 1

$3x + 2y = 12$

$3(2) + 2(3) = 12$

$6 + 6 = 12$

$12 = 12$

TRUE

Equation 2

$2x - y = 1$

$2(2) - 3 = 1$

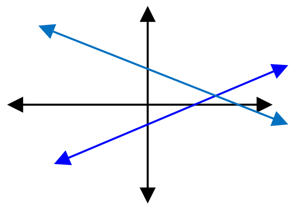
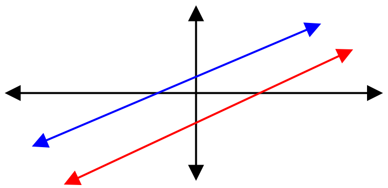
$4 - 3 = 1$

$1 = 1$

TRUE

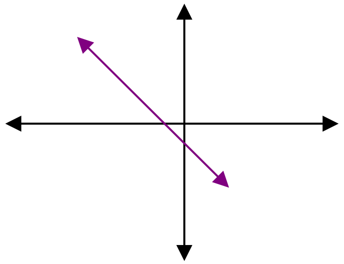
Point (2,3) is the solution of the given system of equations.

Objective 3:
Use graphing to identify systems with no solution or infinitely many solutions.

Number of Solutions to a System of Two Linear Equations		
Number of Solutions	What This Means Graphically	Graphical Examples
Exactly one ordered-pair solution	The two lines intersect at one point.	
No solution	The two lines are parallel.	

Objective 3:

Use graphing to identify systems with no solution or infinitely many solutions.

Number of Solutions to a System of Two Linear Equations *(continued)*		
Number of Solutions	What This Means Graphically	Graphical Examples
Infinitely Many Solutions	The two lines are identical.	

Objective 3:
Use graphing to identify systems with no so or infinitely many solutions.

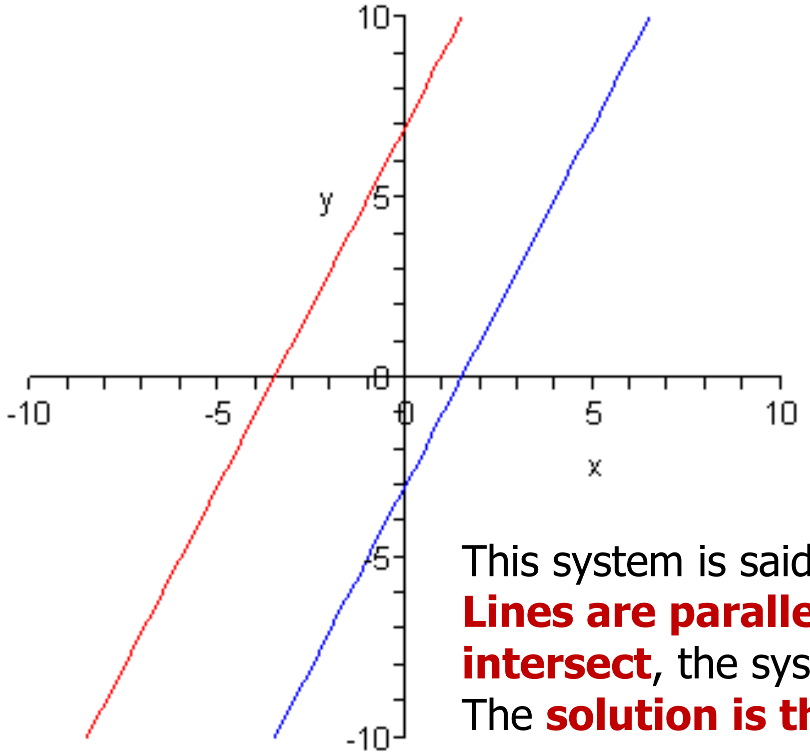
Example 1: Solve the system by graphing:

$$y = 2x - 3$$
$$y = 2x + 7$$

Note that both equations have the same slope (m = 2) but different y-intercepts.

First Line: $y = 2x - 3$
Slope = 2
Y-Intercept = -3; Line passes through (0,-3).
To graph, we start at the *y*-intercept and move 2 up (the rise) and 1 right (the run).

Second Line: $y = 2x + 7$
Slope = 2
Y-Intercept = 7; Line passes through (0,7).
To graph, we start at the *y*-intercept and move 2 up (the rise) and 1 right (the run).



This system is said to be **inconsistent**. **Lines are parallel** and **fail to intersect**, the system has no solution. The **solution is the empty set**.

Objective 3:
Use graphing to identify systems with no so or infinitely many solutions.

Example 2: Solve the system by graphing:

$$x + y = 3$$
$$2x + 2y = 6$$

1. Graph the first line. Find x and y intercepts of **x + y = 3**.

X – Intercept (let y = 0)

$x + y = 3$
$x + 0 = 3$
$x = 3$

Y – Intercept (let x = 0)

$x + y = 3$
$0 + y = 3$
$y = 3$

The x-intercept is 3, while the y-intercept is 3.

Objective 3:
Use graphing to identify systems with no so or infinitely many solutions.

Example 2: Solve the system by graphing:

$$x + y = 3$$
$$2x + 2y = 6$$

2. Graph the second line. Find x and y intercepts of **2x + 2y = 6**.

X – Intercept (let y = 0)

$2x + 2y = 6$
$2x + 2(0) = 6$
$2x = 6$
$x = 3$

Y – Intercept (let x = 0)

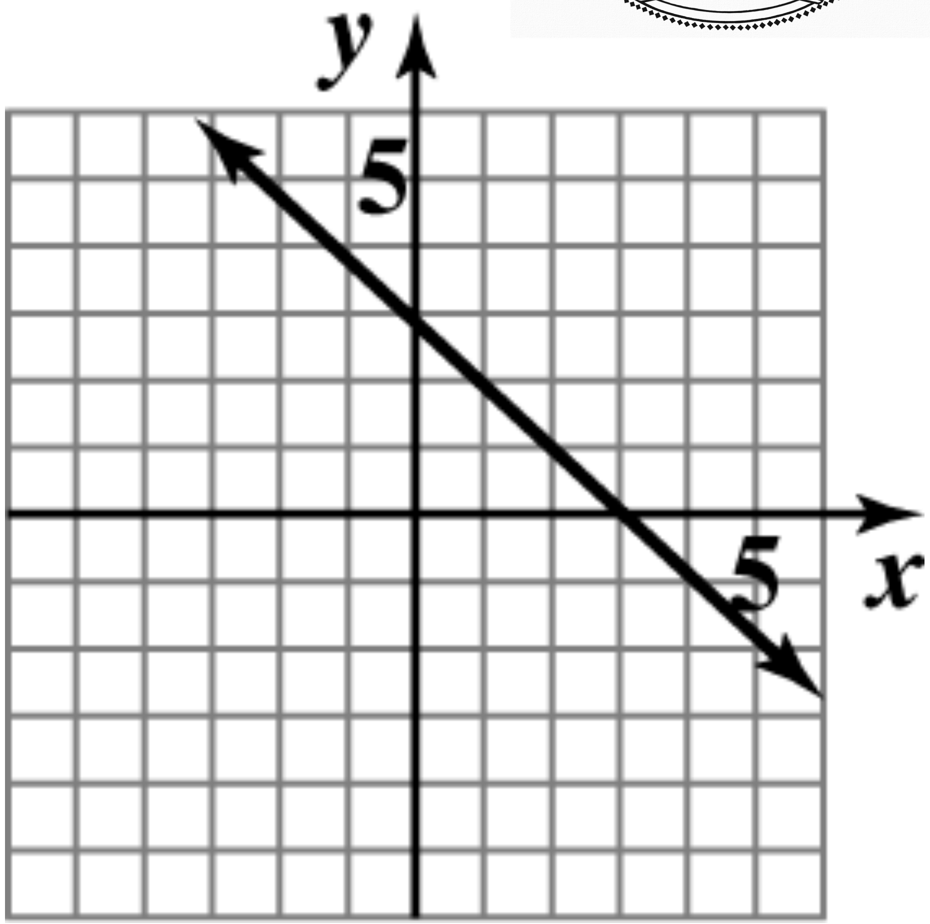
$2x + 2y = 6$
$2(0) + 2y = 6$
$2y = 6$
$y = 3$

The x-intercept is 3, while the y-intercept is 3.

Objective 3:
Use graphing to identify systems with no so or infinitely many solutions.

Example 2: Solve the system by graphing:

$$x + y = 3$$
$$2x + 2y = 6$$



Both lines have the same *x*- and y-intercepts.

Thus, the graphs of the two equations in the system are the same line.

The system has an infinite number of solutions, namely all points that are solutions of either line.

Solution Set: $\{(x, y) | x + y = 3\}$.

OBJECTIVES:

1. Determine whether an ordered pair is a solution of a linear system. ✓
2. Solve systems of linear equations by graphing. ✓
3. Use graphing to identify systems with no solution or infinitely many solutions. ✓